INNOVATION AND IMITATION WITH AND WITHOUT INTELLECTUAL PROPERTY RIGHTS

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Abstract. An extensive empirical literature indicates that returns from innovation are appropriated primarily via mechanisms *other* than formal intellectual property rights – and that 'imitation' is itself a costly activity. However theory has tended tended to assume the pure nonrivalry of 'ideas' with its implication that in the absence of intellectual property rights innovation (and welfare) is zero. This paper introduces a model of innovation based on imperfect competition in which imitation is costly. We demonstrate that in the absence of intellectual property rights a significant proportion of innovation may still occur, and that welfare may be higher in the absence of intellectual property rights even though less innovation occurs.

Keywords: Innovation, Imperfect Competition, Intellectual Property, Imitation

JEL codes: K3, L5, O3

1. Introduction

Repeated surveys, including Levin et al. (1987), Mansfield (1985), Cohen, Nelson, and Walsh (2000), and Arundel (2001), show that firms appropriate returns from innovation using a variety of methods including secrecy, lead time, marketing and sales, learning curve advantages and patents. Furthermore, they also suggest that for most industries (with a few notable exceptions) patent protection is of low importance. As Hall (2003) summarizes (p. 9): 'In both the United States and Europe, firms rate superior sales and service, lead time, and secrecy as far more important than patents in securing the returns to innovation. Patents are usually reported to be important primarily for blocking and defensive purposes.'

Of particular interest is the finding that imitation is a costly process and one, furthermore, upon which the effect of a patent – if it has any effect at all – is to increase its cost

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not to halt it entirely. Perhaps most striking in this respect are Tables 8 (p. 810) and 9 (p. 811) of Levin et al. (1987) which summarize, respectively, reported cost of imitation (as a percentage of innovator's R&D expenditure) and time to imitate. *Even without patent protection*, for major unpatented processes 43% of firms said that imitation cost was between 51 and 75% of innovator R&D while 39% said that it was between 75 and 100% (a further 6% said that imitation cost was more than 100% or impossible). For major unpatented products the analogous figures were 46%, 31% and 9%. In terms of imitation lag a similar pattern emerges. For major unpatented processes the lag is 1 to 3 years in 66% of cases and longer than that in a further 18% cases (it is less than 6 months only 2% of the time). For major unpatented products these figures were 70% and 12% and 2%[1].

Such results indicate that for many innovations, even without patent protection, imitation involves substantial cost and delay(it should be emphasized that the distinction between innovation and imitation is often highly blurred and that imitation itself is a creative process. Given this, as well as the strong impact the assumption of costless imitation has on our conclusions, it would seem important to investigate the consequences of weakening this presumption and, in particular, the possibilities of innovation without intellectual property rights.

However most of the existing theoretical literature has tended to assume 'perfect' non-rivalry, that is, that an innovation (or creative work) once made may be costlessly, and instantaneously, reproduced. The assumption is most often evident in the claim, which follows directly from it, namely that without the provision of intellectual property rights such as patents and copyrights no innovation would be possible.

For example, Nordhaus (1969) (and following him Scherer, 1972), in what is considered to be one of the founding papers of the policy literature, implicitly assume that without a patent an innovator gains no remuneration. Similarly, Klemperer (1990) in his paper on patent breadth makes clear his assumption of costless imitation[2](p. 117): 'For simplicity, I assume free entry into the industry subject to the noninfringement of the patent and that

[1]Of course, the problems of selection bias loom large here. Patents are sought precisely for those products where imitation is easy and, conversely, when imitation is difficult patents will be less valuable and therefore used less. What is really required are estimates of imitation cost for the same innovation both with and without patent protection.

[2]Though it should be noted that it is possible to interpret the travel cost incurred by consumers in Klemperer's model as some form of 'design-around' or imitation cost that must be paid by competing firms. Nevertheless in Klemperer's model without IP the innovator's profits post entry are zero and therefore makes a net loss (and hence would not enter).

knowledge of the innovation allows competitors' products to be produced *without* fixed costs and at the same constant marginal costs as the patentholder's product. Without further loss of generality, I assume marginal costs to be zero.' (Emphasis added). Many similar examples can easily be supplied in which imitation without IPRs is 'trivial'.

Some papers in the literature do allow non-trivial imitation, for example Gallini (1992) and Waterson (1990). However, imitation (and its associated cost) is not of primary theme. Waterson (1990), for example, has entry by a competing (imitative) firm within a horizontal product differentiation framework and focuses on the impact of patent breadth on litigation and (socially valuable) product differentiation.

This paper, by contrast, provides a theoretical model based on Stackelberg competition in which costly imitation is central and non-trivial amounts of innovation occur in the absence of intellectual property rights. As well as allowing us to compare the relative performance of regimes with and without intellectual property rights, the model also allows us to clearly distinguish the sources of welfare differences between the two regimes: 1) less innovation occurs without intellectual property rights; 2) that the set of innovations occurring without intellectual property rights systematically deliver more welfare on average than those that do not; 3) the welfare of a given innovation is higher under competition that under monopoly. We illustrate these effects using a simple uniform of innovation, showing that, compared to a regime with intellectual property rights, a regime without them, while **having only half the level of innovation provides three quarters of the welfare**.

2. The Model

The model is based on the Stackelberg model of quantity competition with multiple followers. In our case, the first mover role is naturally taken by the developer of the original innovation who we term the 'innovator', and the role of followers by 'imitators'. All firms are the same except for the fact that the innovator has different fixed costs from those of imitators. There is no formal delay in innovation but the Stackelberg framework implicitly assumes the first-mover has time enough to commit to supply as much of the market as she wishes. Demand is taken to be linear with an inverse demand curve $p(q) = a - bq$. To summarize:

(1) f_i the fixed cost of development for the innovator.
(2) f_m the fixed cost of imitation which is assumed to be common across all imitators. Also define ϕ to be imitation cost as a proportion of innovation cost. We assume that imitation cost is always less than innovation cost and that in the presence of intellectual property rights imitation does not occur (which could be interpreted as having infinite imitation cost). Note that while greatly simplifying the analysis this does not fit with the empirical data from Levin et al discussed above (we will return to this issue below).
(3) $c(q)$, marginal cost of production once the product is developed. It is assumed to be common between imitators and innovators (they both end up using the same technology), to be constant, and, without loss of generality, to be equal to zero.
(4) Linear demand given by $p(q) = a - bq$

We have a slight variation on the classic two-stage model in which the sequence of actions can be considered as falling into three periods as follows:

(1) An innovator decides whether to enter. If the innovator does enter then (s)he incurs a fixed cost, f_i, and develops a new product
(2) Imitators decide whether to enter. If an imitator does enter (s)he incurs a fixed cost of f_m, and then has capacity to produce the new product.
(3) Production occurs with price and quantities determined by Stackelberg competition in which the 'innovator' has the first-mover role and all imitators move simultaneously.

A Normalization: Define $k = \frac{a^2}{4b}$ so k is equal to half the area under the demand curve and therefore the level of monopoly profit. No agent's profits (innovator or imitator) can be greater than monopoly profits k. Hence let us simplify by normalizing all profits and fixed costs by dividing them by k – equivalent to setting k equal to 1 in the analysis below. Thus from now on when profits or fixed costs are discussed they should be taken not as absolute levels but as proportions of monopoly profits (itself equal to half of total potential welfare offered by the innovation).

Remark: The space of innovations. In this model an innovation is specified by the tuple (f_i, f_m). Under the assumptions given and using normalized variables the space of innovations is then $IS = (f_i, f_m) \in [0,1] \times [0,1] : f_m \leq f_i = (f_i, \phi) \in [0,1] \times [0,1]$. Finally let g be the density function giving the distribution of innovations over this space.

Remark: the effect of intellectual property rights. Our starting point is always the distribution, g, of innovations just described above. The introduction of intellectual property rights (patent or copyright) can then be modelled in one of two ways:

(1) All imitation is prohibited (i.e. the number of imitators is restricted to 0).
(2) The original distribution g is transformed to a new distribution g'

In what follows we shall, by default, proceed using the first case and assume that patents do not alter the distribution of innovation but instead simply prevent imitators from entering (of course, in one sense we can interpret the first situation as simply a special case of the second in which imitation cost is infinite).

3. Solving the Model

Proposition 1. *Let n be the number of imitators. The solution to the Stackelberg model of competition by quantify is as follows ('I' subscripts are on variables related to the innovator and 'imm' subscripts are on variables related to an imitator):*

$$q_I = \frac{a}{2b} \tag{3.1}$$

$$q_{imm} = \frac{a}{2b(n+1)} \tag{3.2}$$

$$\textit{Total output} = Q = \frac{a(2n+1)}{2b(n+1)} \tag{3.3}$$

$$p = a - bQ = \frac{a}{2(n+1)} \tag{3.4}$$

$$\textit{Gross profits of an innovator} = \Pi_I = \frac{k}{n+1} = \frac{1}{n+1} \tag{3.5}$$

$$\textit{Gross profits for an imitator} = \Pi_{imm} = \frac{k}{(n+1)^2} = \frac{\Pi_I}{n+1} \tag{3.6}$$

$$\tag{3.7}$$

Proof. Omitted (the solution to the standard Stackelberg model is well-known). □

Proposition 2. *Allowing the number of imitators to take non-integer values then the set of innovations which occur is given by:*

$$A = \{(f_i, f_m) \in IS : f_m \geq f_i^2\}$$

Proof. Imposing a zero net profit condition on the basis of free entry as an imitator gives the number of imitators, n^e:

$$\Pi_{imm} = f_m \Rightarrow n^e = \sqrt{\frac{k}{f_m}} - 1$$

Innovation only occurs if expected (net) profits are positive, that is $\Pi_I \geq f_i$. Substituting for the LHS using our value for the number of innovators gives the condition:

$$f_m \geq f_i^2$$

□

Proposition 3. *Restricting the number of imitators to integer values the set of innovations that occur is:*

$$A = \cup_{n=0}^{\infty}\{(f_i, f_m) \in IS : \frac{1}{n^2} \geq f_m > \frac{1}{(n+1)^2}, f_i \leq \frac{1}{n+1}\}$$

Proof. With integer n the expected number of imitators, n^e, will be the largest n such that $(n+1)^2 f_m \leq 1$. □

Remark 1. Note the substantial difference between the two situations (non-integer and integer numbers of imitators). For example, $f_m \geq \frac{1}{4} \Rightarrow n = 0$ and all innovations with $f_i \leq 1$ are realized, a very different outcome to that with continuous n.

Corollary 4. *With intellectual property rights all innovations,* $(f_i, f_m) \in IS$*, occur.*

Proof. We have assumed that with intellectual property rights no imitation is possible, that is $n = 0$. Substituting this into the working of Proposition 2 and 3 yields in both cases $A = (f_i, f_m) \in IS$ □

In this model an innovation is defined by a pair (f_i, f_m) giving its innovation and imitation cost. We can therefore visualise potential innovations in a two dimensional graph of innovation/imitation cost space. In particular, we can summarize the results of the previous propositions in Figure 1. In this diagram the light-shaded (yellow) region is

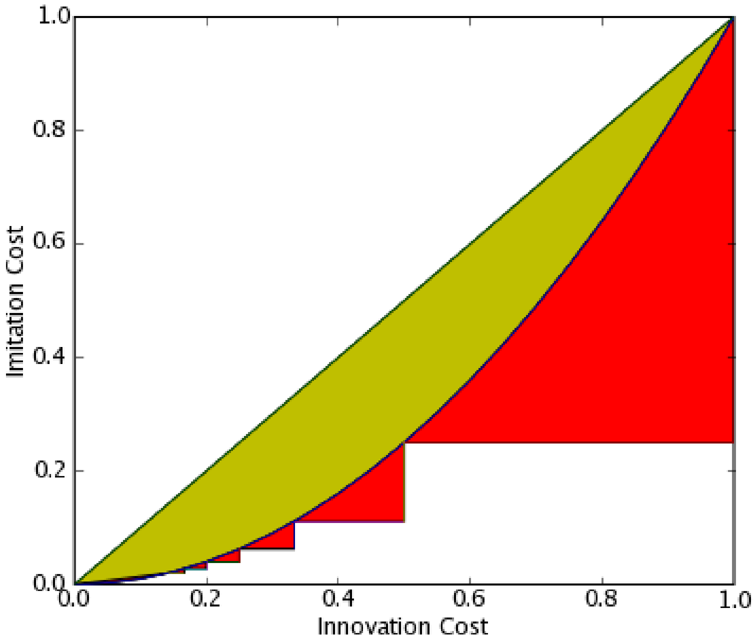

FIGURE 1. Innovation, imitation cost pairs for which innovation occurs without intellectual property rights

that in which innovations occur with non-integer numbers of imitators permitted, while the innovations in the dark-shaded (red) and light-shaded region occur when restricting to integer numbers of imitators. (The region above the diagonal should be ignored since we are assuming that imitation cost is always less than innovation cost).

While the preceding diagram is entirely correct as it stands, it will be useful to visualize the same data in a slightly different manner. We do this by replacing imitation cost by 'proportional' imitation cost (ϕ) – i.e. imitation cost as a proportion of innovation cost. Under our assumption that imitation cost is always less that innovation cost this means that we now have a constant range, [0,1], for 'proportional' imitation cost at all levels of innovation cost and, in visual terms, we have a uniform level of innovation per unit of innovation cost. This is shown in Figure 2 which is simply a re-rendering of Figure 1 using proportional innovation cost.

4. Innovation and Welfare

4.1. **Innovation.** First we consider the level of innovation without intellectual property rights in the case of a uniform distribution of potential innovations.

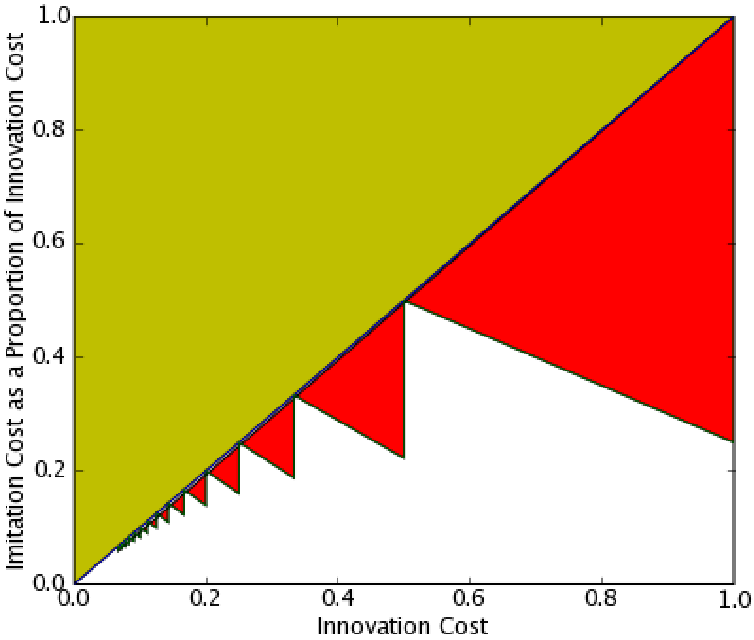

FIGURE 2. Innovation, proportional imitation cost pairs for which innovation occurs without intellectual property rights

Proposition 5. *Assuming a uniform distribution over the space of innovations shown in Figure 2, that is with density function $g(f_i, \phi) = 1$, the ratio of innovation without intellectual property rights to that with intellectual property rights is: 50% (non-integer n), 72% (integer n).*

Proof. See appendix. □

4.2. **Welfare.** From a policy perspective what really matters is the utility generated by innovation not how much innovation occurs. If the welfare from innovations realized without intellectual property rights differ systematically from those that are not (as is the case) then welfare proportions will be different from innovation proportions.

Let ZI denote the regime with zero imitation (in which case all innovation takes place). Let R be another regime and let A be the region in which innovation takes place under R. Furthermore define $B = IS - A$, that is, the set of innovations not in A. Define:

$$W^R(f_i,\phi) = \text{Welfare under regime R from innovation}(f_i,\phi) \tag{4.1}$$

$$\Delta W^R(f_i,\phi) = W^R(f_i,\phi) - W^{ZI}(f_i,\phi) \tag{4.2}$$

$$W_X^R = \text{Welfare from region X under regime R} = \int_X W^R(f_i,f_m) \tag{4.3}$$

$$\Delta W_X^R = W_X^R - W_X^{ZI} \tag{4.4}$$

Then:

$$W^{ZI} = W_A^{ZI} + W_B^{ZI} \tag{4.5}$$

$$W^R = W_A^R + W_B^R = W_A^{ZI} + \Delta W_A^R + 0 \tag{4.6}$$

The second equation illustrates how we may break up the welfare under a given regime. The welfare from region B, W_B^R is zero since, by definition, no innovation occurs in that region.

Then we may divide welfare from region A into the welfare we would get in the case of zero imitation (the first term) plus the difference between that level and the level of welfare in regime R: ΔW.

This allows us to distinguish between two effects that operate with respect to differences in welfare when imitation is permitted. First, is the fact, already mentioned, that innovation fixed costs may differ systematically between regions A and B. This will materialize in the relative sizes of W_A and W_B.

Second is the effect encapsulated in the ΔW term which captures the fact that, for a *given innovation*, the welfare generated from it as affected by whether imitation is permitted or not. These differences in turn derive from two sources. First, an increase in consumer surplus with imitation since total output is higher and price lower than with without imitation. Second, a reduction in producer surplus with imitation due to lower prices and larger fixed costs due to the greater number of producers.

Proposition 6. *The difference in welfare generated under regime R compared to the zero imitation regime is always non-negative and its precise value is:*

$$\Delta W^R(f_i, f_m) = \frac{n^2}{2(n+1)^2} \geq 0$$

Proof. See appendix. □

Proposition 7. *Assuming a uniform distribution over the space of innovations as shown in Figure 2, that is with density function $g(f_i, \phi) = 1$, welfare levels are as follows (where NIP indicates a regime without intellectual property rights and the number of imitators may take non-integer values):*

$$W_A^{ZI} = \tfrac{7}{12}, \textit{average welfare density} = \tfrac{7}{6} \tag{4.7}$$

$$W_B^{ZI} = \tfrac{5}{12}, \textit{average welfare density} = \tfrac{5}{6} \tag{4.8}$$

$$\Delta W_A^{NIP} \approx \tfrac{2}{12}, \textit{average welfare density} = \tfrac{2}{6} \tag{4.9}$$

Proof. See appendix. □

Thus, the ratio of welfare without intellectual property rights to a situation in which they are present is **75%**, i.e. without intellectual property rights while only having half the level of innovation we have three quarters of the welfare achieved with intellectual property rights. While the result here is specific to the assumption on the distribution of innovations the underlying point that welfare proportions will always be systematically higher than innovation proportions (even if we ignore deadweight loss) holds in general.

Proposition 8. *The ratio of welfare achieved with and without intellectual property rights will be greater than or equal to the ratio of innovation with and without intellectual property rights with this inequality being strict if the level of innovation without intellectual property rights is non-zero.*

Proof. See appendix. □

Proposition 9. *Assuming a uniform distribution of proportional imitation costs if f_i is less than 0.2 in an industry then welfare is higher without intellectual property rights in that industry (f_i is innovation cost as a proportion of potential monopoly profits).*

Proof. See appendix. □

Proposition 10. *Assuming a uniform distribution of innovation costs if the proportional imitation cost for an industry is greater than 0.8 then welfare is higher without intellectual property rights.*

Proof. See appendix. □

5. Conclusion

In this paper we have presented simple model of innovation with imitation. We have shown that when imitation is costly even though the initial innovator does not enjoy a pure monopoly she may still be able to garner sufficient rents to cover the fixed cost of development.

Here innovations are specified by a tuple consisting of the 'innovation' cost and the 'imitation' cost (the innovation cost being the cost to the first developer of the product). Settling on two simple uniform-type distributions of innovations over this space we calculated the level of innovation and the level of welfare with and without intellectual property rights.

Using as our metric the ratio of innovation (welfare) without intellectual property rights to the situation where they are present we derived several significant results. First, that the ratio for welfare was higher than that for innovation. The reason for this was that the innovations that do occur without intellectual property rights deliver, on average, higher welfare than those that do not occur. Second, while the level of innovation without intellectual property rights must always be lower than that with (at least in the model as presented here) this need not be true of welfare. In particular we found in one case that while the ratio of innovation was only 50% the ratio of welfare was 75%.

The introduction of a framework with which to properly model imitation is doubly useful. First, it squares far better with the empirical literature. Second, it serves to show that the assumption of pure nonrivalry is not an innocent one. On the first point it is worth reiterating that existing research clearly demonstrates not only that imitation is both costly and time consuming but that the effect of patent, rather than being to deliver a true monopoly, is to increase the effort required for imitation. If this is so then any attempt to adequately model the process of innovation (and to base policy thereon) must commence from a model based on imitation and imperfect competition rather than one based on the assumption of pure monopoly.

6. Further Work

There are a varieties of way in which the work presented could be extended. One could, for example, introduce a 'race' for the innovation in standard manner. This would allow for multiple firms at the innovation stage competing to produce the original innovation. This could be extended so that failed innovators can be imitators at the second stage.

On a separate point, one distinctive feature of this model is that intellectual property rights always lead to maximal innovation. In a more complex model, for example one involving cumulative innovation, this might no longer be the case. There are a variety of approaches that could be taken to integrate such dynamics and investigating these options would be one of most important improvements to the model that could be made.

Another option, which was mentioned in the introduction, was to model imitation delay as well as imitation cost. Similarly, allowing for types of imperfect competition other than Stackelberg would also be a valuable extension. For example, the models of Waterson (1990) and Klemperer (1990) both provide for product differentiation and these models could be adapted to provide a richer and more realistic model of imitation in the presence – and absence – of intellectual property rights.

Appendix A. Proofs of Propositions

Proof of Proposition 5. A uniform distribution of innovation corresponds to the standard euclidean measure over IS, which in turns corresponds to calculating areas in Figure 2. With intellectual property rights no imitation is permitted so all the innovations in the figure occur (total area of the figure is 1). Thus to calculate the proportions of innovation occurring without intellectual property rights we need to calculate the size of the dark-shaded and light-shaded areas as proportion of the entire figure.

For continuous n we consider the light-shaded region. This, clearly, has area equal to 1/2.

Restricting to integer n we need to add to this the area of the dark-shaded (red) region. The area of the dark-shaded (red) region is made up of a series of similar triangles. The nth triangle (working down from the largest) has area:

$$0.5 \cdot b \cdot h = 0.5 \cdot (\frac{1}{n} - \frac{1}{n+1}) \cdot (\frac{1}{n} - \frac{n}{(n+1)^2})$$

Thus total area of dark-shaded (red) region is:

$$0.5 \sum_1^{\infty} (\frac{1}{n} - \frac{1}{n+1}) \cdot (\frac{1}{n} - \frac{n}{(n+1)^2}) = 0.5 \cdot (\sum \frac{1}{n^2} - \sum \frac{1}{n(n+1)} - \sum \frac{1}{(n+1)^2} + \sum \frac{n}{(n+1)^3})$$

All of these sums are simple except for the third. For this one approximate as follows:

$$\sum \frac{n}{(n+1)^3} \approx \sum_1^{99} \frac{n}{n+1}^3 + \int_{99}^{infty} \frac{1}{(x+1)^2} = 0.432976 + 0.01 = 0.4430$$

Substituting this gives the dark-shaded (red) region's total area as:

$$0.5 \cdot (\sum \frac{1}{n^2} - \sum \frac{1}{n(n+1)} - \sum \frac{1}{(n+1)^2} + \sum \frac{n}{(n+1)^3}) = 0.5 \cdot ((1+X) - 1 - X + 0.4430) = 0.2215$$

Thus total area of light-shaded and dark-shaded region is $0.5 + 0.2215 \approx 0.72$.

□

Proof of Proposition 6. First let us determine the welfare arising from a given innovation. If there are n imitators we have that consumer surplus (CS) and producer surplus (PS) are as follows:

$$CS(f_i, f_m) = 0.5 \cdot (a-p) \cdot q = \tfrac{(2n+1)^2}{2(n+1)^2} \quad \text{(A.1)}$$

$$PS(f_i, f_m) = \Pi_I - f_i + n \cdot (\Pi_{imm} - f_m) = \tfrac{1}{n+1} - f_i \quad \text{(A.2)}$$

Note that we have used the fact that, with continuous n, the zero profit condition implies $\Pi_{imm} = f_m$. Summing to get total welfare we have:

$$W(f_i, f_m) = CS + PS = \frac{(2n+1)^2}{2(n+1)^2} + \frac{1}{n+1} - f_i$$

Now in a ZI regime $n = 0$ so:

$$W^{ZI} = \frac{3}{2} - f_i$$

Thus,

$$\Delta W(f_i, f_m) = W^R(f_i, f_m) - W^{ZI}(f_i, f_m) \tag{A.3}$$

$$= (\tfrac{(2n+1)^2}{(n+1)^2} + \tfrac{1}{n+1} - f_i) - (\tfrac{3}{2} - f_i) \tag{A.4}$$

$$= \tfrac{n^2}{2(n+1)^2} \tag{A.5}$$

□

Proof of Proposition 7. To calculate total welfare for region X we integrate welfare per innovation, $W(f_i, f_m)$, over X.

$$W_A^{ZI} = \frac{1}{2}(\frac{3}{2} - \text{avg over A}(f_i)) = \frac{3}{4} - \frac{1}{2}\frac{1}{3} = \frac{7}{12}$$

$$W_B^{ZI} = \frac{1}{2}(\frac{3}{2} - \text{avg over B}(f_i)) = \frac{3}{4} - \frac{1}{2}\frac{2}{3} = \frac{5}{12}$$

Calculating ΔW is slightly more complicated:

$$\Delta W_A = \int_A \frac{n^2}{2(n+1)^2} = \int_0^1 \int_{f_i}^1 d\phi df_i$$

Recall that:

$$\phi = \frac{f_m}{f_i} \tag{A.6}$$

$$n + 1 = \frac{1}{\sqrt{f_m}} \Rightarrow \frac{n^2}{(n+1)^2} = 1 - 2\sqrt{f_m} + f_m \tag{A.7}$$

Thus, substituting f_m for ϕ as well as for n we have:

$$\Delta W_A = 0.5 \int_0^1 \frac{1}{f_i} \int_{f_i^2}^{f_i} 1 - 2\sqrt{f_m} + f_m df_m df_i$$

Working through the first integration gives:

$$\Delta W_A = 0.5 \int_0^1 1 - \frac{4\sqrt{f_i}}{3} - \frac{f_i}{2} + \frac{4f_i^2}{3} - \frac{f_i^3}{2} df_i = \frac{13}{72} \approx \frac{1}{6}$$

□

Proof of Proposition 9. We need to determine welfare at a particular level of f_i (innovation cost as a proportion of potential monopoly profit) assuming a uniform distribution of proportional imitation cost under an IP (zero imitation) and no IP regime. Proceeding as above but making all welfare calculations a function of f_i we have:

$$W_A^{ZI}(f_i) = (\tfrac{3}{2} - f_i)(1 - f_i) \tag{A.8}$$

$$WB^{ZI}(f_i) = (\tfrac{3}{2} - f_i)f_i \tag{A.9}$$

$$\Delta W_A(f_i) = \tfrac{1}{2}(1 - \tfrac{4\sqrt{f_i}}{3} - \tfrac{f_i}{2} + \tfrac{4f_i^2}{3} - \tfrac{f_i^3}{2} \tag{A.10}$$

The difference in welfare between a regime without IP compared to one with is $\Delta W(f_i) = W^{NIP}(f_i) - W^{ZI}(f_i)$. Thus to determine the cut-off point, α say, such that for all $f_i \leq \alpha$ the no IP regime is preferable we simply need to solve:

$$\Delta W(f_i) = 0$$

(Note that ΔW is a decreasing function of f_i so the solution will be unique and that $\Delta W(0) > 0$ and $\Delta W(1) < 0$ so a solution will exist).

Proceeding numerically we obtain a figure of $\alpha = 0.191 \approx 0.2$. □

Proof of Proposition 10. By symmetry between ϕ and f_i (see Figure 2) the results for the previous proposition apply here but with the cutoff point for proportional imitation cost equal to $1 - \alpha$. □

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